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Modal component mode synthesis in torsional vibration analysis: rotor-blade interaction

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Abstract

For safe rotor operation it is important to predict natural frequencies of the full rotor arrangement and not only its components. These natural frequencies are typically speed-dependent if rotor and blade vibrations are coupled. In this contribution we focus on the torsional rotor-blade interaction, the coupling between torsional vibrations of the shaft and bending vibrations of blade rows attached to the shaft.

During the design of a turbine shaft train, rotor blades are modelled using 3D finite elements due to its complex geometry and resulting vibration modes. This kind of model incorporates typically centrifugal loading due to the rotor rotation as well as contact modelling at the rotor-blade interface. Building a 3D finite element model describing the rotor-blade interaction can become cumbersome and its calculation is time consuming. Instead of modelling the whole shaft train in 3D finite elements, the proposed modelling is based on the simple 1D rotor beam model. Employing the method of substructuring enables to translate any complex blade which is modelled using 3D finite elements with thousands of physical degrees of freedom into a bunch of models with a single modal degree of freedom described by its natural frequencies and the so-called modal effective moments of inertia.

It is outlined how the natural frequencies of a rotor with flexible blades are predicted by coupling the modal properties of two or more subsystems: the rotor with rigid blades and the isolated flexible blade rows. The resulting model resembles the rotor-blade interaction in all its details from the rotor point of view. It has to be highlighted that his process allows incorporating several modes of any blade row to be incorporated in the final rotordynamic model.

1 Introduction

For simple torsional rotordynamic models, it is sufficient to model the blade rows by rigid inertias which are directly attached to the rotor. This modelling implicitly neglects the speed-dependent blade stiffening due to centrifugal load as well as the dynamic coupling between the torsional vibrations of the shaft and the bending vibrations of the blade rows [1,3]. As soon as these effects are pronounced, a more detailed modelling is necessary which incorporates the flexibility of the blade row.

During the design of an Alstom gas or steam turbine shaft train, rotor blades are modelled using 3D finite elements due to its complex geometry and resulting vibration modes. This kind of model incorporates typically centrifugal loading due to the rotor rotation as well as contact modelling at the rotor-blade interface [2,3]. Building a complete 3D finite element model to describe the rotor-blade interaction can become cumbersome and its calculation is time consuming. In general, a rotor does not need to be modelled in such a detail. If its axisymmetric geometry is exploited a 1D representation of the rotor in terms of its torsional shaft elements is sufficient and accurate.

Instead of modelling the entire shaft train in 3D finite elements, the proposed modelling is based on the simple 1D rotor shaft model. Employing the method of substructuring enables to translate any complex blade which is modelled using 3D finite elements with thousands of physical degrees of freedom into a bunch of models with a single modal degree of freedom. Natural frequencies and modal masses are assigned to each modal degree of freedom representing the blade vibrations. These single degree of freedom models are coupled via so-called modal effective moments of inertia to the rotor shaft model. The structure of such a rotordynamic model and the resulting accuracy in the prediction of the natural frequencies are outlined in more detail in [12]. This procedure allows even an efficient numerical implementation and prediction of mistuned blade dynamics.

In the following sections, a well-known substructuring method is briefly outlined which is especially suitable for the analysis of rotor-blade interaction. It is outlined how the natural frequencies of a rotor with flexible blades are predicted by coupling the modal properties of two or more subsystems: the rotor with rigid blades and the isolated flexible blade rows. The method is examined on a simple gas turbine shaft train.

2 Model reduction and modal substructuring

The finite element model of a vibrating blade is defined by a mass and a stiffness matrix. Solving for dynamic properties or responses generally involves an inverse matrix operation or an eigenvalue problem. The computational cost associated with these mathematical operations is disproportionately high compared to the number of degrees of freedom involved. A complete 3D finite element model of a turbine rotor including all blade rows may consist of up to several millions degrees of freedom. From the rotodynamic point of view, this leads in general to excessive computation time and prevents or slows down design iterations. In order to significantly reduce the computing time associated with rotordynamics while keeping the accuracy on an acceptable level, a model reduction is necessary.

In the following a model reduction and substructuring method of built-up structures is outlined which is especially suitable for the analysis of rotor-blade coupling. This kind of model reduction techniques can be used to reduce the computational cost drastically which is especially important for the analysis of the full rotor arrangement. A reduction in model size can imply a loss of information and the reduced model then only gives an approximation to the solution provided by the full model. Therefore the reduction method should be case-specific and truncate only unwanted, unimportant or redundant information. An approach that goes hand in hand with reduction techniques is substructuring and subsequent component mode synthesis (CMS) [4]. The development of CMS methods started with a publication by Hurty in 1965 [5]. Since then, numerous CMS methods have been presented of which some are documented in [3,6-11].

If a structure is divided into several components, the computational cost of solving all smaller problems independently is in general less than solving the original large problem [7]. The full system could be divided into components (substructures) in an arbitrary fashion, e.g. defined according to mathematical properties rather than physical or geometric boundaries. For rotor-blade interaction, the components *rotor* and *blades* are designed and developed with different scopes and by different groups so it is natural to define the blade row as a single component which is rigidly attached to the main rotor structure and is interfaced to rotordynamics. Therefore, from all the available component mode synthesis (CMS) methods, the fixed-interface Craig-Bampton method [9] is the most appropriate choice for the analysis of rotor-blade coupling.

2.1 Fixed-interface component mode synthesis for rotor-blade coupling

The fixed-interface CMS method couples components rigidly by employing a specific coordinate transformation and applying a reduction in size at component level in the modal space before assembling the reduced component to the main system. The static and dynamic behaviour of each component is described in terms of a set of basis functions. These include normal modes found from solving a component eigenvalue problem and additional static constraint modes. The reduction in size is achieved by truncating higher frequency modes at the component level. The implication of fixed-interface CMS is visualised in Fig. 1.

The dynamics of a complex blade that is fixed at the rotor interface is described by its degrees of freedom $\mathbf{q}$ and the stiffness and inertia matrices $\mathbf{K}$ and $\mathbf{M}$. A coordinate transformation is applied on the physical DOFs leading to a new set of component matrices

$$\mathbf{q} = \mathbf{T}_{CB}\,\mathbf{u}: \qquad \mathbf{M}_{CB} = \mathbf{T}_{CB}^{T}\,\mathbf{M}\,\mathbf{T}_{CB}, \quad \mathbf{K}_{CB} = \mathbf{T}_{CB}^{T}\,\mathbf{K}\,\mathbf{T}_{CB} \tag{1}$$

Herein, CB stands for the abbreviation for Craig-Bampton.

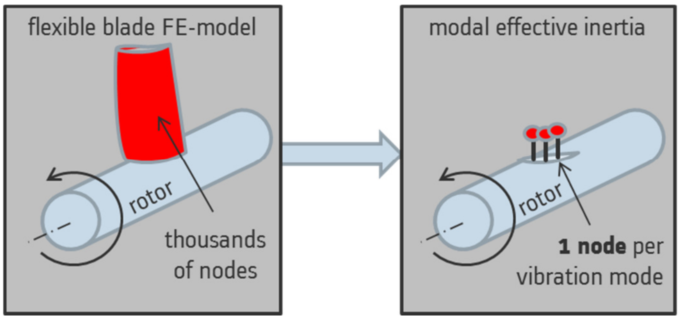

Fig. 1. Visualisation of component mode synthesis in the context of rotor-blade interaction (taken from [12])

The main requirement is that the coordinate transformation is reversible, i.e. the transformation matrix has full rank. This is achieved for the fixed-interface CMS method by choosing the two linearly independent sets of modes: the *fixed interface modes* and the *static constraint modes*. The component degrees of freedom are divided into two sets: the interior ($\mathbf{q}_i$) and the boundary DOFs ($\mathbf{q}_b$). The component matrices are partitioned according to these two sets into

$$\mathbf{M} = \left[\begin{array}{c|c} \mathbf{m}_{ii} & \mathbf{m}_{ib} \\ \mathbf{m}_{ib^T} & \mathbf{m}_{bb} \end{array}\right], \qquad \mathbf{K} = \left[\begin{array}{c|c} \mathbf{k}_{ii} & \mathbf{k}_{ib} \\ \mathbf{k}_{ib^T} & \mathbf{k}_{bb} \end{array}\right] \tag{2}$$

The fixed interface modes are the resulting component modes with all boundary degrees of freedom fixed. For mass-normalised modes these read

$$\boldsymbol{\varphi}_j^T\ \mathbf{m}_{ii}\ \boldsymbol{\varphi}_j = 1, \quad \boldsymbol{\varphi}_j^T\ \mathbf{k}_{ii}\ \boldsymbol{\varphi}_j = \omega_j^2, \quad j = 1, 2, \ldots \tag{3}$$

The static constraint modes are static deflections also known from Guyan reduction (static condensation) [9] which result from a unit displacement at one boundary DOF while all other boundary DOFs are fixed,

$$\boldsymbol{\psi} = -\mathbf{k}_{ii}^{-1}\,\mathbf{k}_{ib} \tag{4}$$

These two mode sets define a coordinate transformation with full rank for fixed-interface CMS with respect to the partitioned component coordinates $\mathbf{q}$

$$\begin{gathered} \mathbf{T}_{CB} = \left[\begin{array}{c|c} \boldsymbol{\Phi} & \boldsymbol{\Psi} \\ \mathbf{0} & \mathbf{1} \end{array}\right] : \quad \mathbf{K}_{CB} = \left[\begin{array}{c|c} \boldsymbol{\omega}^2 & \mathbf{0} \\ \mathbf{0}^T & \mathbf{k}_{ib}^T\boldsymbol{\Psi} + \mathbf{k}_{bb} \end{array}\right] \\ \mathbf{M}_{CB} = \left[\begin{array}{c|c} \mathbf{1} & \boldsymbol{\Phi}^T\mathbf{m}_{ii}\boldsymbol{\Psi} + \boldsymbol{\Phi}^T\mathbf{m}_{ib} \\ \text{sym.} & \boldsymbol{\Psi}^T\mathbf{m}_{ii}\boldsymbol{\Psi} + \boldsymbol{\Psi}^T\mathbf{m}_{ib} + \mathbf{m}_{ib}^T\boldsymbol{\Psi} + \mathbf{m}_{bb} \end{array}\right] \end{gathered} \tag{5}$$

Herein, $\boldsymbol{\omega}^2$ denotes a diagonal matrix of the eigenvalues of the fixed component. At this state only the lowest fixed interface modes are kept. If n is the number of kept blade row modes, then the size of diagonal eigenvalue matrix becomes ***n*** x ***n***. The modally reduced flexible blade model is attached to the main rotor structure at the component boundary DOFs by assembling the system matrices of the main rotor structure (superscript r) with the rigidly attached reduced blade component matrices in Eq. (5),

$$\mathbf{K}^{\text{sys}} = \begin{bmatrix} \mathbf{k}_{ii}^{\text{r}} & \mathbf{k}_{ib}^{\text{r}} & \mathbf{0} \\ \mathbf{k}_{bi}^{\text{r}} & \mathbf{k}_{bb}^{\text{r}} + \mathbf{k}_{bb} & \mathbf{k}_{bi} \\ \mathbf{0} & \mathbf{k}_{bi} & \boldsymbol{\omega}^2 \end{bmatrix}, \quad \mathbf{M}^{\text{sys}} = \begin{bmatrix} \mathbf{m}_{ii}^{\text{r}} & \mathbf{m}_{ib}^{\text{r}} & \mathbf{0} \\ \mathbf{m}_{bi}^{\text{r}} & \mathbf{m}_{bb}^{\text{r}} + \mathbf{m}_{bb} & \mathbf{m}_{bi} \\ \mathbf{0} & \mathbf{m}_{bi} & \mathbf{1} \end{bmatrix} \tag{6}$$

with respect to the coordinates $\mathbf{q} = \left[\mathbf{q}_i^{\mathrm{r},T}, \mathbf{q}_b^{\mathrm{r},T} = \mathbf{q}_b^T, \mathbf{q}_i^T\right]^T$. Since a turbine shaft train consists of several flexible blades, this procedure is repeated for each blade row that needs to be considered flexible in the rotordynamic assessment.

The main advantages of the described procedure are summarized here:

- The procedure is dynamically exact in the frequency range of interest.
- The resulting size of the full rotor dynamic model becomes small enough to allow elaborate parametric studies and design optimisations.
- Translating the complex 3D blade model in to a bunch of simple oscillators can be extracted in an automated fashion from 3D finite element software packages, e.g. ANSYS or ABAQUS. This is less error prone than the commonly applied manual slicing of blades to generate a lumped mass model.
- Defined interfaces facilitates cross-discipline collaboration and allow fast as well as targeted iteration in the design tuning which are standard in development projects.

2.2 Modal representation of the blade

If we consider only torsional DOFs at the main rotor structure then the physical boundary DOF at the interface to the flexible blade row reduces to a single angular displacement. For simplicity, only the first two modes of the flexible blade row are considered. The extension to several modes is straightforward. For this drastic reduction, the component matrices in Eq. (5) reduce to

$$\mathbf{M}_{CB} = \left[\begin{array}{cc|c} 1 & 0 & \Gamma_1 \\ 0 & 1 & \Gamma_2 \\ \hline \Gamma_1 & \Gamma_2 & \Theta_r \end{array}\right], \quad \mathbf{K}_{CB} = \left[\begin{array}{cc|c} \omega_1^2 & 0 & 0 \\ 0 & \omega_2^2 & 0 \\ \hline 0 & 0 & 0 \end{array}\right] \tag{7}$$

Herein, Θ_r denote the moment of inertia of the rigid blade row with respect to the rotor axis and Γ_j represent the modal participation factors. Associated with the participation factor are the *modal effective moment of inertia* (MEMOI) which are defined as Γ_j^2. The first two columns of these matrices correspond to the first two modal coordinates of the fixed blade row while the last column corresponds to the physical rotation of the rotor disc. The component described by the matrices in Eq. (7) corresponds to the system as initially sketched in Fig. 1 on the right hand side.

The system matrices in Eqs. (6) and (7) are dynamically equivalent leading to the same natural frequencies. Each blade mode is condensed to a physical 1DOF system having inertia of the size of the corresponding MEMOI which is attached by a torsional spring to the rotor interface. This resembles the flexible part of the blade. On top of that Eq. (7) allows a further interpretation that the rigid part of the blade, rigid in the sense of natural frequencies being sufficiently far from the frequency range of interest, is attached directly to the rotor interface after subtracting the fractions that are considered as flexible.

2.3 Natural frequencies of the coupled rotor-blade system by means of a modal approach

An eigenvalue problem for the coupled rotor-blade system can be derived by using the natural frequencies and mode shapes of the two decoupled systems and built up the coupled equations by means of a modal transformation. Starting with the *decoupled* subsystems, two independent sets of natural frequencies, mode shapes (eigenvectors), generalized masses and generalized stiffnesses are known by solving the eigenvalue problem for each subsystem. In the following we assume that all eigenvectors are mass-normalised according to Eq. (3), however, the outlined procedure can be easily generalised to an arbitrary scaling of the eigenvectors. The subsystems in the context of rotor-blade coupling are:

- Torsional rotor system including the inertia terms of the rigid blades fixed to the rotor:
  $\omega_{T,l}$, γ_{kl}, $\boldsymbol{\gamma}_l$ … *l*th natural frequency, eigenvector component at blade row *k* and full rotor mode *l*

- Flexible blades (bending), fixed at interface:
  $\omega_{B,j}$, Γ_j, $\boldsymbol{\varphi}_j$ … *j*th natural frequency, MEMOI and eigenvector of blade mode *j*

From the eigenvalue problem for the blades, the MEMOI is easily determined and represents the coupling parameter for the two subsystems. Usually the above modal properties are derived by the eigenvalue solver of finite element programs.

By means of a modal transformation, the vectors of the blade displacements $\mathbf{u}(t)$ and the rotational displacements $\boldsymbol{\psi}(t)$ of the rotor are expressed as

$$\mathbf{u}(t) = \sum_{j=1}^{N} \eta_j(t)\,\boldsymbol{\varphi}_j \quad \text{eigenvectors of decoupled blade modes } \boldsymbol{\varphi}_j \tag{8}$$

$$\boldsymbol{\psi}(t) = \sum_{l=1}^{M} \alpha_l(t)\,\boldsymbol{\gamma}_l \quad \text{torsional eigenvectors of decoupled rotor modes } \boldsymbol{\gamma}_l \tag{9}$$

These transformations are introduced into the equations of motion of the coupled rotor-blade system leading to a (N+M)x(N+M) eigenvalue problem.

A special case of this eigenvalue problem is given if only one eigenvector is used from each eigenvector set, e.g. $\boldsymbol{\varphi}_j$ and $\boldsymbol{\gamma}_l$. This results in two coupled equations of motions from which an eigenvalue problem for the two generalised coordinates η_j and α_l is formulated as shown in the following equation. This eigenvalue problem represents an approximation of eigenvalue problem of the full system.

$$\begin{bmatrix} \omega_{T,l}^2 - \omega_{lj}^2 & -\omega_{lj}^2 \gamma_{kl} \Gamma_j \\ -\omega_{lj}^2 \gamma_{kl} \Gamma_j & \omega_{B,j}^2 - \omega_{lj}^2 \end{bmatrix} \begin{bmatrix} \hat{\alpha}_l \\ \hat{\eta}_j \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{10}$$

ω_{lj} … is the natural frequency of the coupled rotor-blade system (lth rotor mode with ith blade mode),
$\omega_{T,l}$ … is the lth torsional natural frequency of the rotor with rigid blades attached to the rotor,
$\omega_{B,j}$ … is the jth natural frequency of the blade fixed at the rotor (no coupling with rotor),
Γ_j … is the MEMOI of the jth blade mode which depends on the blade eigenvectors, the modal mass and the blade geometry,
γ_{kl} … is the kth entry of the lth rotor eigenvector $\boldsymbol{\gamma}_l$ where k is the location of the blade row.

The term $\gamma_{kl}\Gamma_j$ in the 2x2 eigenvalue problem is a measure for the coupling strength. If this term is zero or relatively small then the combination of the natural frequency $\omega_{T,l}$ of the modal subsystem rotor with rigid blades with the natural frequency $\omega_{B,j}$ of the modal subsystem of the isolated blade mode will not lead to new natural frequencies ω_{lj} in the coupled system. By using the simple 2x2 eigenvalue problem we can now investigate all combinations of $\omega_{T,l}$ (l=1,2,…,N) and $\omega_{B,j}$ (j=1,2,…,M) and look for new natural frequencies in the coupled system.

3 Application to gas turbines

A simple example is presented for outlining the proposed procedure: an Alstom gas turbine shaft train consisting of a gas turbine, an intermediate shaft, a generator and an exciter shaft. Only one blade row, the turbine last stage blade row, is considered to be flexible for the present discussion although the number of flexible blade rows is not restricted. The first few torsional mode shapes of the rotor with the rigid as well as the flexible blade row are calculated with a finite element model and are shown in Fig. 2.

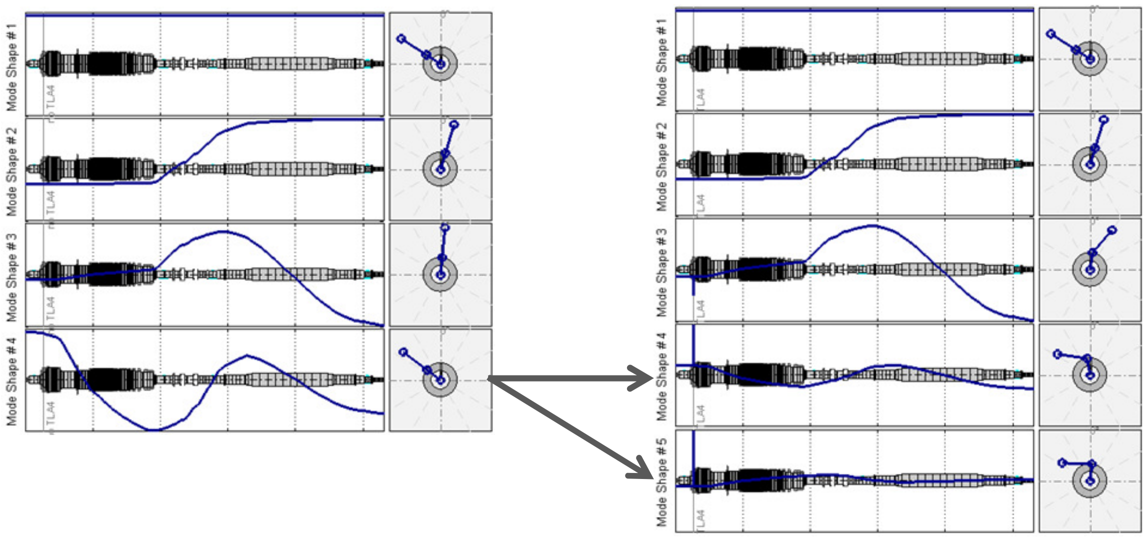

Fig. 2. Torsional mode shapes of the full finite element model of the flexible rotor with a (left) rigid and (right) flexible turbine blade row. The plots show the modal angular displacements along the rotor and in more detail at the blade disc.

The modal properties of the rotor with the rigid blade row in Fig. 2 on the left hand side are combined with the modal properties (including the MEMOI) of the fixed blade row by means of the 2x2 eigenvalue problem defined in Eq. (8). A comparison between the “exact” prediction coming from a finite element calculation and the simplified predictions resulting from the 2x2 eigenvalue problem is summarised in Fig. 3. The first 15 natural frequencies of the full rotor-blade system are predicted with an accuracy of less than 0.2% relative error.

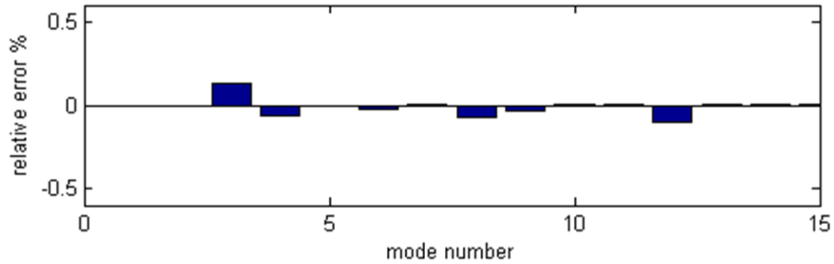

Fig. 3. Relative error in the first 15 natural frequencies between the predicted 2x2 eigenvalue problem in Eq. (8) and the natural frequencies calculated by a finite element model.

A more detailed comparison of the distribution of the natural frequencies of the rotor-blade system is given in Fig. 4 via a natural frequency diagram as very well known in dynamics (see e.g. [3]). In this plot all solutions of the 2x2 eigenvalue problem in Eq. (10). For a weak coupling $\gamma_{kl}\Gamma_j$ between one rotor and one blade mode, solving the eigenvalue problem is not necessary. If it is solved anyway, the resulting natural frequencies lie on the straight lines in Fig. 4. For a strong coupling $\gamma_{kl}\Gamma_j$, the natural frequencies of the coupled modes are different to the natural frequencies of the individual subsystems, see ω_B/ω_T = 1.08. This situation corresponds to the modes 4 and 5 in Fig. 2.

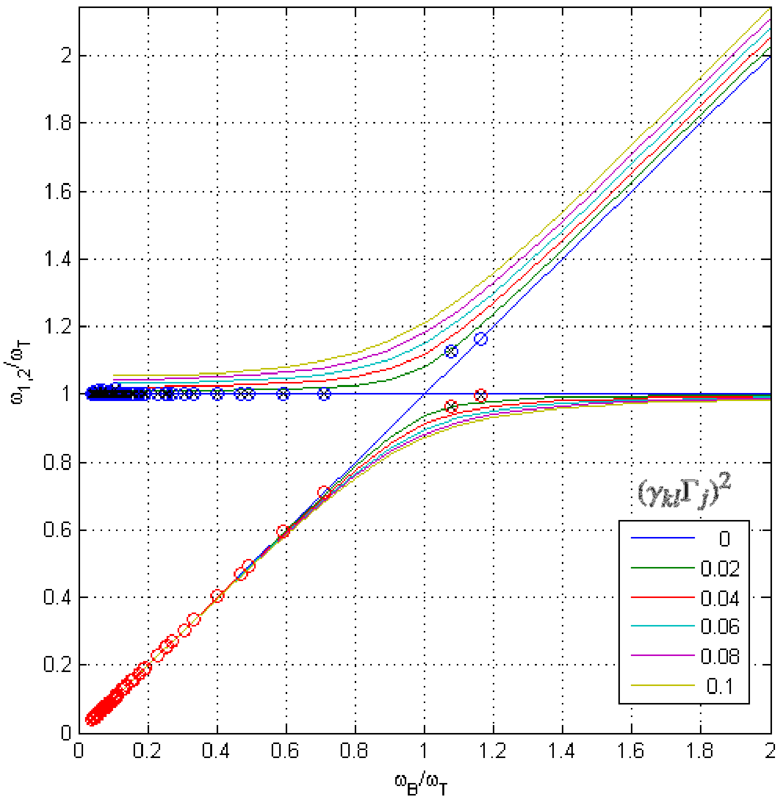

Fig. 4. Rotor-blade interaction for example gas turbine: (solid lines) location of natural frequencies for in dependency of the coupling strength, (circles) both solutions of the 2x2 eigenvalue problem and (crosses) result from the full finite element model.

4 Conclusion

A substructure technique is described that incorporates blade dynamics into the rotordynamic analysis. The presented method is integrated seamlessly in the assessment process of Alstom's shaft trains. It enables and facilitates the collaboration of the different disciplines as the data generation and data exchange is automated and straightforward. This leads to a fast as well as targeted iteration in the design tuning which are standard in development projects.

The resulting model resembles the rotor-blade interaction in all its details from the rotor point of view. It has to be highlighted that the representation of the reduced model is exact compared to the 3D finite element model as long as all modes in the frequency range of interest are considered. There is no approximation of the rotor-blade coupling involved in this process. On top of this, this process allows to incorporate several modes of any blade row to be included in the final rotor dynamic model. Each blade mode adds one node to the model so the impact on computation time for a rotordynamic assessment is negligible.

The efficiency of this process is two-fold. On one hand, the resulting model size of the full rotor dynamic model becomes small and simple enough to allow elaborate parametric studies and design optimisations. On the other hand, translating the complex 3D blade model in to a bunch of single degrees of freedom oscillators is extracted straightforwardly from commercial finite element software packages like Abaqus or Ansys.

References

[1] Ehehalt, U., Becs, B., Zhou, X. and Güllenstern S. (2013): Speed dependency of coupled rotor-blade torsional natural frequencies. In *Proceedings of ASME Turbo Expo 2013: Turbine Technical Conference and Exposition*, San Antonio, Texas, USA, June 3-7, GT2013-95804, pp. 1-8.

[2] Tsypkaykin, I. and Kellerer, R. (2013): Full rotordynamic modelling approach for assessment of dynamic stresses on rotor and blades due to torsional disturbances. In *Proceedings of ASME Turbo Expo 2013*, San Antonio, Texas, USA, June 3-7, GT2013-94855, 10 pages.

[3] Gasch, R., Knothe, K., and Liebich, R. (2012): *Strukturdynamik*. Springer Vieweg, 2nd edition.

[4] Sunar, M. (2002): *Encyclopedia of Vibration*. Vol. 3, Academic Press, pp. 1332–1335

[5] Hurty, W. C. (1965): Dynamic analysis of structural systems using component modes. *AIAA Journal* **3**, pp. 678–685.

[6] Thomson, W. (1981): *Theory of Vibration with Applications*. 2nd edition, Prentice Hall, New Jersey.

[7] Meirovitch, L. (1980): *Computational Methods in Structural Dynamics*. Sijthoff and Noordhoff, Alphen aan der Rijn, Netherlands.

[8] Guyan, R. J. (1965): Reduction of stiffness and mass matrices. *AIAA Journal* **3(2)**, pp. 380.

[9] Craig, R. R., J. and Bampton, M. (1968): Coupling of substructures for dynamic analysis. *AIAA Journal* **6**, pp. 1313–1319.

[10] Craig, R. R., J. (1977): Methods of component mode synthesis. *Shock and Vib. Digest* **9**, pp. 3–10.

[11] Craig, R. R., J. (1981): *Structural Dynamics – An Introduction to Computer Methods*. Wiley.

[12] Dohnal, F., Knopf, E. and Nordmann, R. (2014): Efficient modelling of rotor-blade interaction using substructuring. In *Proceedings of 9th IFToMM International Conference on Rotor Dynamics (ICORD)*, Milan, Italy, September 22-25, 10 pages.

[13] Bachschmid, N., Salvini, G., Tanzi, E. and Pesatori, E. (2014): The Influence of Blade Row Dynamics on Lateral and Torsional Shaft Vibrations in Steam Turbines. In *Proceedings of 9th IFToMM International Conference on Rotor Dynamics (ICORD)*, Milan, Italy, September 22-25, 15 pages.